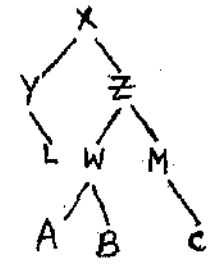
Department of Mathematics
Indian Institute of Technology Kharagpur
MA 21007 Design & Analysis of Algorithms (3 - 1 - 0)
Mid-Autumn 2010 Time: 2 hrs. Marks : 30
Depth : II yr. MA; Breadth: other departments

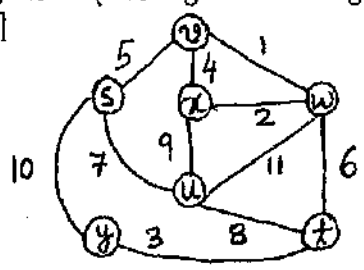
Note : Answer all the questions. Please do not write the algorithm unless it is asked. The terminology and function notation is as discussed in the class. Marks corresponding to each question are shown in bold face at the end of respective question.

1. Fill in the following blanks [1 + 1 + 2]
(a) Given a list of n elements, to delete the maximum from that list, the complexity of the best algorithm considering the number of comparisons also in view, in the worst case is ————————
(b) Suppose that there are n distinct elements in a binary maxheap. Which positions out of (a) 2 or 3, (b) 4 or 5 only, (c) 4 or 5 or 6 only, (d) 4 or 5 or 6 or 7 ————————
(c) If the concept of an essentially complete binary tree is extened to an essentially complete k-ary tree, where $k \geq 3$, the children of a node loacted at $T(i)$ are ———————— and the the upper bound for the height of this k-ary tree in terms the number of elements n is ————————
2. Prove or disprove that $f + g \in \Theta(max(f, g))$ where $f, g : N \to R^+$ are arbitrary functions. [2]
3. Let G be an undirected graph with edge costs C_e, and $C_d = C_e + 1$. Let T be a MST of G and S be a shortest simple path from a node s to another node t, where both T and S are with respect to C_e. Then prove or disprove that (a) T is still a MST with respect to the edge costs C_d, (b) S is still a shortest path from s to t with respect to the edge costs C_d. [4]

4 . Write the procedure "left rotate" which is used to rotate an unbalanced tree. Use the procedure to left rotate the given tree.[3]



5. Consider the given graph G. Give the first four edges that Prim's algorithm (starting from s) adds, in the order they are added. Also, give the first four edges that Dijkstra's algorithm (starting from s and going to t) adds, in the order they are added. [3]



6. In order to compute a^{23} the usual method needs 22 multiplications. Can you propose a divide and conquer method that involve squarrings and multiplications which are fewer than the usuall method, considering that squarring is also a multiplication. How many multiplications are needed using your method to compute a^{23}. [2]

7 . Given two sets A and B each of size n, and a number x, give an algorithm to find a pair of elements, one from A and one from B, whose sum adds up to x. Of course a $\Theta(n^2)$ algorithm is possible, but you are supposed to do better than that. Give full justification. [4]

8. Adjacency matrix of an undirected graph is given. Write an algorithm which finds the number of connected components of the graph using $find(a)$ and $merge(a, b)$ functions of disjoint set structures. Analyze the complexity of your algorithm and find an upper bound as tight as possible. [4]

9. Suppose that you have a row of n coins, with values v1, v2, . . . , vn, all of which are non-negative. You are allowed to pick up any subset of those coins you want, so long as you pick up no two adjacent ones. Your goal is to pick up as large a total value as possible. For example, if the values from left to right were (1, 8, 6, 3, 6), the best you could do would be to pick up the 2nd and 5th coins, for a total value of 14. If the values were (3, 4, 3, 4, 3), the best would be to pick up the 1st, 3rd and 5th coins, for a total value of 9. You are asked to derive a polynomial-time greedy algorithm for finding the most valuable legal subset. Analyze the complexity of your algorithm. [4]

****Good Luck***